

PARI-GP Reference Card

(PARI-GP version 2.3.0)

Note: optional arguments are surrounded by braces {}.

Starting & Stopping GP

to enter GP, just type its name: `gp`
to exit GP, type `\q` or `quit`

Help

describe function `?function`
extended description `??keyword`
list of relevant help topics `???pattern`

Input/Output & Defaults

output previous line, the lines before `%, %', %'', etc.`
output from line n `%n`
separate multiple statements on line `;`
extend statement on additional lines `\`
extend statements on several lines `{seq1; seq2;`
comment `/* ... */`
one-line comment, rest of line ignored `\\ ...`
set default d to val `default({d},{val},flag)`
mimic behaviour of GP 1.39 `default(compatible,3)`

Metacommands

toggle timer on/off `#`
print time for last result `##`
print $%n$ in raw format `\a n`
print $%n$ in pretty format `\b n`
print defaults `\d`
set debug level to n `\g n`
set memory debug level to n `\gm n`
enable/disable logfile `\l {filename}`
print $%n$ in pretty matrix format `\m`
set output mode (raw, default, prettyprint) `\o n`
set n significant digits `\p n`
set n terms in series `\ps n`
quit GP `\q`
print the list of PARI types `\t`
print the list of user-defined functions `\u`
read file into GP `\r filename`
write $%n$ to file `\w n filename`

GP Within Emacs

to enter GP from within Emacs: `M-x gp, C-u M-x gp`
word completion `(TAB)`
help menu window `M-\c`
describe function `M-?`
display $\mathrm{T\!E\!X}$ 'd PARI manual `M-x gpman`
set prompt string `M-\p`
break line at column 100, insert `M-\\`
PARI metacommand `\letter` `M-\letter`

Reserved Variable Names

$\pi = 3.14159\dots$ `Pi`
Euler's constant $= .57721\dots$ `Euler`
square root of -1 `I`
big-oh notation `O`

PARI Types & Input Formats

<code>t_INT</code> . Integers	$\pm n$
<code>t_REAL</code> . Real Numbers	$\pm n.ddd$
<code>t_INTMOD</code> . Integers modulo m	<code>Mod(n, m)</code>
<code>t_FRAC</code> . Rational Numbers	n/m
<code>t_COMPLEX</code> . Complex Numbers	$x + y * I$
<code>t_PADIC</code> . p -adic Numbers	$x + O(p^k)$
<code>t_QUAD</code> . Quadratic Numbers	$x + y * \text{quadgen}(D)$
<code>t_POLMOD</code> . Polynomials modulo g	<code>Mod(f, g)</code>
<code>t_POL</code> . Polynomials	$a * x^n + \dots + b$
<code>t_SER</code> . Power Series	$f + O(x^k)$
<code>t_QFI/t_QFR</code> . Imag/Real bin. quad. forms	<code>Qfb($a, b, c, \{d\}$)</code>
<code>t_RFRAC</code> . Rational Functions	f/g
<code>t_VEC/t_COL</code> . Row/Column Vectors	$[x, y, z], [x, y, z] \sim$
<code>t_MAT</code> . Matrices	$[x, y, z, t; u, v]$
<code>t_LIST</code> . Lists	<code>List($[x, y, z]$)</code>
<code>t_STR</code> . Strings	<code>"aaa"</code>

Standard Operators

basic operations	<code>+, -, *, /, ^</code>
<code>i=i+1, i=i-1, i=i*j, ...</code>	<code>i++, i--, i*=j, ...</code>
euclidean quotient, remainder	$x \backslash y, x \backslash y, x \% y, \text{divrem}(x, y)$
shift x left or right n bits	$x << n, x >> n$ or <code>shift(x, n)</code>
comparison operators	<code><=, <, >=, >, ==, !=</code>
boolean operators (or, and, not)	<code> , &&, !</code>
sign of $x = -1, 0, 1$	<code>sign(x)</code>
maximum/minimum of x and y	<code>max, min(x, y)</code>
integer or real factorial of x	$x!$ or <code>factorial(x)</code>
derivative of f w.r.t. x	f'

Conversions

Change Objects

to vector, matrix, set, list, string	<code>Col/Vec, Mat, Set, List, Str</code>
create PARI object ($x \bmod y$)	<code>Mod(x, y)</code>
make x a polynomial of v	<code>Pol($x, \{v\}$)</code>
as above, starting with constant term	<code>Polrev($x, \{v\}$)</code>
make x a power series of v	<code>Ser($x, \{v\}$)</code>
PARI type of object x	<code>type($x, \{t\}$)</code>
object x with precision n	<code>prec($x, \{n\}$)</code>
evaluate f replacing vars by their value	<code>eval(f)</code>

Select Pieces of an Object

length of x	<code>#x</code> or <code>length(x)</code>
n -th component of x	<code>component(x, n)</code>
n -th component of vector/list x	$x[n]$
(m, n) -th component of matrix x	$x[m, n]$
row m or column n of matrix x	$x[m,], x[, n]$
numerator of x	<code>numerator(x)</code>
lowest denominator of x	<code>denominator(x)</code>

Conjugates and Lifts

conjugate of a number x	<code>conj(x)</code>
conjugate vector of algebraic number x	<code>conjvec(x)</code>
norm of x , product with conjugate	<code>norm(x)</code>
square of L^2 norm of vector x	<code>norml2(x)</code>
lift of x from Mods	<code>lift, centerlift(x)</code>

Random Numbers

random integer between 0 and $N - 1$ `random( $\{N\}$ )`
get random seed `getrand()`
set random seed to s `setrand( $s$ )`

Lists, Sets & Sorting

sort x by k th component `vecsort( $x, \{k\}, \{fl = 0\}$ )`
Sets (= row vector of strings with strictly increasing entries)
intersection of sets x and y `setintersect( $x, y$ )`
set of elements in x not belonging to y `setminus( $x, y$ )`
union of sets x and y `setunion( $x, y$ )`
look if y belongs to the set x `setsearch( $x, y, flag$ )`
Lists
create empty list of maximal length n `listcreate( $n$ )`
delete all components of list l `listkill( $l$ )`
append x to list l `listput( $l, x, \{i\}$ )`
insert x in list l at position i `listinsert( $l, x, i$ )`
sort the list l `listsort( $l, flag$ )`

Programming & User Functions

Control Statements (X : formal parameter in expression seq)
eval. seq for $a \leq X \leq b$ `for( $X = a, b, seq$ )`
eval. seq for X dividing n `fordiv( $n, X, seq$ )`
eval. seq for primes $a \leq X \leq b$ `forprime( $X = a, b, seq$ )`
eval. seq for $a \leq X \leq b$ stepping s `forstep( $X = a, b, s, seq$ )`
multivariable for `forvec( $X = v, seq$ )`
if $a \neq 0$, evaluate seq_1 , else seq_2 `if( $a, \{seq_1\}, \{seq_2\}$ )`
evaluate seq until $a \neq 0$ `until( $a, seq$ )`
while $a \neq 0$, evaluate seq `while( $a, seq$ )`
exit n innermost enclosing loops `break( $\{n\}$ )`
start new iteration of n th enclosing loop `next( $\{n\}$ )`
return x from current subroutine `return( $x$ )`
error recovery (try seq_1) `trap( $\{err\}, \{seq_2\}, \{seq_1\}$ )`

Input/Output

prettyprint args with/without newline `printp(), printp1()`
print args with/without newline `print(), print1()`
read a string from keyboard `input()`
reorder priority of variables x, y, z `reorder( $\{[x, y, z]\}$ )`
output $args$ in $\mathrm{T\!E\!X}$ format `printtex( $args$ )`
write $args$ to file `write, write1, writetex( $file, args$ )`
read file into GP `read( $\{file\}$ )`

Interface with User and System

allocates a new stack of s bytes `allocatemem( $\{s\}$ )`
execute system command a `system( $a$ )`
as above, feed result to GP `extern( $a$ )`
install function from library `install( $f, code, \{gpf\}, \{lib\}$ )`
alias old to new `alias( $new, old$ )`
new name of function f in GP 2.0 `whatnow( $f$ )`

User Defined Functions

`name(formal vars) = local(local vars); seq`
`struct.member = seq`
kill value of variable or function x `kill( $x$ )`
declare global variables `global( $x, \dots$ )`

Iterations, Sums & Products

numerical integration `intnum( $X = a, b, expr, flag$ )`
sum $expr$ over divisors of n `sumdiv( $n, X, expr$ )`
sum $X = a$ to $X = b$, initialized at x `sum( $X = a, b, expr, \{x\}$ )`
sum of series $expr$ `suminf( $X = a, expr$ )`
sum of alternating/positive series `sumalt, sumpos`
product $a \leq X \leq b$, initialized at x `prod( $X = a, b, expr, \{x\}$ )`
product over primes $a \leq X \leq b$ `prodeuler( $X = a, b, expr$ )`
infinite product $a \leq X \leq \infty$ `prodinf( $X = a, expr$ )`
real root of $expr$ between a and b `solve( $X = a, b, expr$ )`

Vectors & Matrices

dimensions of matrix x	<code>matsize(x)</code>
concatenation of x and y	<code>concat($x, \{y\}$)</code>
extract components of x	<code>vecextract($x, y, \{z\}$)</code>
transpose of vector or matrix x	<code>mattranspose(x)</code> or <code>x-</code>
adjoint of the matrix x	<code>matadjoin(x)</code>
eigenvectors of matrix x	<code>mateigen(x)</code>
characteristic polynomial of x	<code>charpoly($x, \{v\}, flag$)</code>
minimal polynomial of x	<code>minpoly($x, \{v\}$)</code>
trace of matrix x	<code>trace(x)</code>

Constructors & Special Matrices

row vec. of $expr$ eval'd at $1 \leq i \leq n$	<code>vector($n, \{i\}, \{expr\}$)</code>
col. vec. of $expr$ eval'd at $1 \leq i \leq n$	<code>vectorv($n, \{i\}, \{expr\}$)</code>
matrix $1 \leq i \leq m, 1 \leq j \leq n$	<code>matrix($m, n, \{i\}, \{j\}, \{expr\}$)</code>
diagonal matrix whose diag. is x	<code>matdiagonal(x)</code>
$n \times n$ identity matrix	<code>matid(n)</code>
Hessenberg form of square matrix x	<code>mathess(x)</code>
$n \times n$ Hilbert matrix $H_{ij} = (i + j - 1)^{-1}$	<code>mathilbert(n)</code>
$n \times n$ Pascal triangle $P_{ij} = \binom{i}{j}$	<code>matpascal($n - 1$)</code>
companion matrix to polynomial x	<code>matcompanion(x)</code>

Gaussian elimination

determinant of matrix x	<code>matdet($x, flag$)</code>
kernel of matrix x	<code>matker($x, flag$)</code>
intersection of column spaces of x and y	<code>matintersect(x, y)</code>
solve $M * X = B$ (M invertible)	<code>matsolve(M, B)</code>
as solve, modulo D (col. vector)	<code>matsolvemod(M, D, B)</code>
one sol of $M * X = B$	<code>matinverseimage(M, B)</code>
basis for image of matrix x	<code>matimage(x)</code>
supplement columns of x to get basis	<code>mat supplement(x)</code>
rows, cols to extract invertible matrix	<code>matindexrank(x)</code>
rank of the matrix x	<code>matrank(x)</code>

Lattices & Quadratic Forms

upper triangular Hermite Normal Form	<code>mathnf(x)</code>
HNF of x where d is a multiple of $\det(x)$	<code>mathnfmod(x, d)</code>
elementary divisors of x	<code>matsnf(x)</code>
LLL-algorithm applied to columns of x	<code>qflll($x, flag$)</code>
like <code>qflll</code> , x is Gram matrix of lattice	<code>qflllgram($x, flag$)</code>
LLL-reduced basis for kernel of x	<code>matkerint(x)</code>
Z -lattice $\longleftrightarrow$ Q -vector space	<code>matrixqz(x, p)</code>
signature of quad form ${}^t y * x * y$	<code>qfsign(x)</code>
decomp into squares of ${}^t y * x * y$	<code>qfgaussred(x)</code>
find up to m sols of ${}^t y * x * y \leq b$	<code>qfminim(x, b, m)</code>
$v, v[i] :=$ number of sols of ${}^t y * x * y = i$	<code>qfrep($x, B, flag$)</code>
eigenvals/eigenvecs for real symmetric x	<code>qfjacobi(x)</code>

Formal & p-adic Series

truncate power series or p -adic number	<code>truncate(x)</code>
valuation of x at p	<code>valuation(x, p)</code>
Dirichlet and Power Series	
Taylor expansion around 0 of f w.r.t. x	<code>taylor(f, x)</code>
$\sum a_k b_k t^k$ from $\sum a_k t^k$ and $\sum b_k t^k$	<code>serconvol(x, y)</code>
$f = \sum a_k * t^k$ from $\sum (a_k / k!) * t^k$	<code>serlaplace(f)</code>
reverse power series F so $F(f(x)) = x$	<code>serreverse(f)</code>
Dirichlet series multiplication / division	<code>dirmul, dirdiv(x, y)</code>
Dirichlet Euler product (b terms)	<code>direuler($p = a, b, expr$)</code>

p-adic Functions

Teichmuller character of x	<code>teichmuller(x)</code>
Newton polygon of f for prime p	<code>newtonpoly(f, p)</code>

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Polynomials & Rational Functions

degree of f	<code>poldegree(f)</code>
coefficient of degree n of f	<code>polcoeff(f, n)</code>
round coeffs of f to nearest integer	<code>round($f, \{&e\}$)</code>
gcd of coefficients of f	<code>content(f)</code>
replace x by y in f	<code>subst(f, x, y)</code>
discriminant of polynomial f	<code>poldisc(f)</code>
resultant of f and g	<code>polresultant($f, g, flag$)</code>
as above, give $[u, v, d], xu + yv = d$	<code>bezoutres(x, y)</code>
derivative of f w.r.t. x	<code>deriv(f, x)</code>
formal integral of f w.r.t. x	<code>intformal(f, x)</code>
reciprocal poly $x^{\deg f} f(1/x)$	<code>polrecip(f)</code>
interpol. pol. eval. at a	<code>polinterpolate($X, \{Y\}, \{a\}, \{&e\}$)</code>
initialize t for Thue equation solver	<code>thueinit(f)</code>
solve Thue equation $f(x, y) = a$	<code>thue($t, a, \{sol\}$)</code>

Roots and Factorization

number of real roots of $f, a < x \leq b$	<code>polsturm($f, \{a\}, \{b\}$)</code>
complex roots of f	<code>polroots(f)</code>
symmetric powers of roots of f up to n	<code>polsym(f, n)</code>
roots of f mod p	<code>polrootsmod($f, p, flag$)</code>
factor f	<code>factor($f, \{lim\}$)</code>
factorization of f mod p	<code>factormod($f, p, flag$)</code>
factorization of f over $\mathbb{F}_{p^a}$	<code>factorff(f, p, a)</code>
p -adic fact. of f to prec. r	<code>factorpadic($f, p, r, flag$)</code>
p -adic roots of f to prec. r	<code>polrootspadic(f, p, r)</code>
p -adic root of f cong. to a mod p	<code>padicappr(f, a)</code>
Newton polygon of f for prime p	<code>newtonpoly(f, p)</code>

Special Polynomials

n th cyclotomic polynomial in var. v	<code>polcyclo($n, \{v\}$)</code>
d -th degree subfield of $\mathbb{Q}(\zeta_n)$	<code>polsubcyclo($n, d, \{v\}$)</code>
n -th Legendre polynomial	<code>pollegendre(n)</code>
n -th Tchebicheff polynomial	<code>pol tchebi(n)</code>
Zagier's polynomial of index n, m	<code>polzagier(n, m)</code>

Transcendental Functions

real, imaginary part of x	<code>real(x), imag(x)</code>
absolute value, argument of x	<code>abs(x), arg(x)</code>
square/ n th root of x	<code>sqrtn($x, n, &z$)</code>
trig functions	<code>sin, cos, tan, cotan</code>
inverse trig functions	<code>asin, acos, atan</code>
hyperbolic functions	<code>sinh, cosh, tanh</code>
inverse hyperbolic functions	<code>asinh, acosh, atanh</code>
exponential of x	<code>exp(x)</code>
natural log of x	<code>ln(x)</code> or <code>log(x)</code>
gamma function $\Gamma(x) = \int_0^\infty e^{-t} t^{x-1} dt$	<code>gamma(x)</code>
logarithm of gamma function	<code>lngamma(x)</code>
$\psi(x) = \Gamma'(x)/\Gamma(x)$	<code>psi(x)</code>
incomplete gamma function ($y = \Gamma(s)$)	<code>incgam($s, x, \{y\}$)</code>
exponential integral $\int_x^\infty e^{-t}/t dt$	<code>eint1(x)</code>
error function $2/\sqrt{\pi} \int_x^\infty e^{-t^2} dt$	<code>erfc(x)</code>
dilogarithm of x	<code>dilog(x)</code>
m th polylogarithm of x	<code>polylog($m, x, flag$)</code>
U -confluent hypergeometric function	<code>hyperu(a, b, u)</code>
J -Bessel function $J_{n+1/2}(x)$	<code>besseljh(n, x)</code>
K -Bessel function of index nu	<code>besselk(nu, x)</code>

Elementary Arithmetic Functions

vector of binary digits of $ x $	<code>binary(x)</code>
give bit number n of integer x	<code>bittest(x, n)</code>
ceiling of x	<code>ceil(x)</code>
floor of x	<code>floor(x)</code>
fractional part of x	<code>frac(x)</code>
round x to nearest integer	<code>round($x, \{&e\}$)</code>
truncate x	<code>truncate($x, \{&e\}$)</code>
gcd/LCM of x and y	<code>gcd(x, y), lcm(x, y)</code>
gcd of entries of a vector/matrix	<code>content(x)</code>

Primes and Factorization

add primes in v to the prime table	<code>addprimes(v)</code>
the n th prime	<code>prime(n)</code>
vector of first n primes	<code>primes(n)</code>
smallest prime $\geq x$	<code>nextprime(x)</code>
largest prime $\leq x$	<code>precprime(x)</code>
factorization of x	<code>factor($x, \{lim\}$)</code>
reconstruct x from its factorization	<code>factorback($fa, \{nf\}$)</code>

Divisors

number of distinct prime divisors	<code>omega(x)</code>
number of prime divisors with mult	<code>bigomega(x)</code>
number of divisors of x	<code>numdiv(x)</code>
row vector of divisors of x	<code>divisors(x)</code>
sum of (k -th powers of) divisors of x	<code>sigma($x, \{k\}$)</code>

Special Functions and Numbers

binomial coefficient $\binom{x}{y}$	<code>binomial(x, y)</code>
Bernoulli number B_n as real	<code>bernreal(n)</code>
Bernoulli vector $B_0, B_2, \dots, B_{2n}$	<code>bernvec(n)</code>
n th Fibonacci number	<code>fibonacci(n)</code>
number of partitions of n	<code>numbpart(n)</code>
Euler ϕ -function	<code>eulerphi(x)</code>
Möbius μ -function	<code>moebius(x)</code>
Hilbert symbol of x and y (at p)	<code>hilbert($x, y, \{p\}$)</code>
Kronecker-Legendre symbol $(\frac{x}{y})$	<code>kronecker(x, y)</code>

Miscellaneous

integer or real factorial of x	<code>x!</code> or <code>fact(x)</code>
integer square root of x	<code>sqrntint(x)</code>
solve $z \equiv x$ and $z \equiv y$	<code>chinese(x, y)</code>
minimal u, v so $xu + yv = \gcd(x, y)$	<code>bezout(x, y)</code>
multiplicative order of x (intmod) (i=0)	<code>znorder($x, \{o\}$)</code>
primitive root mod prime power q	<code>znprimroot(q)</code>
structure of $(\mathbb{Z}/n\mathbb{Z})^*$	<code>znstar(n)</code>
continued fraction of x	<code>contfrac($x, \{b\}, \{lmax\}$)</code>
last convergent of continued fraction x	<code>contfracpnqn(x)</code>
best rational approximation to x	<code>bestappr(x, k)</code>

True-False Tests

is x the disc. of a quadratic field?	<code>isfundamental(x)</code>
is x a prime?	<code>isprime(x)</code>
is x a strong pseudo-prime?	<code>ispseudoprime(x)</code>
is x square-free?	<code>issquarefree(x)</code>
is x a square?	<code>Zissquare($x, \{&n\}$)</code>
is pol irreducible?	<code>polisirreducible(pol)</code>

Based on an earlier version by Joseph H. Silverman

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PARI-GP Reference Card (2)

(PARI-GP version 2.3.0)

Elliptic Curves

Elliptic curve initially given by 5-tuple $E = [a_1, a_2, a_3, a_4, a_6]$. Points are $[x, y]$, the origin is $[0]$.

Initialize elliptic struct. ell , i.e create `ellinit( $E, flag$ )`

$a_1, a_2, a_3, a_4, a_6, b_2, b_4, b_6, b_8, c_4, c_6, disc, j$. This data can be recovered by typing $ell.a1, \dots, ell.j$. If fl omitted, also

E defined over **R**

x -coords. of points of order 2	<code>ell.roots</code>
real and complex periods	<code>ell.omega</code>
associated quasi-periods	<code>ell.eta</code>
volume of complex lattice	<code>ell.area</code>

E defined over $\mathbf{Q}_p, |j|_p > 1$

x -coord. of unit 2 torsion point	<code>ell.roots</code>
Tate's $[u^2, u, q]$	<code>ell.tate</code>
Mestre's w	<code>ell.w</code>

change curve E using $v = [u, r, s, t]$ `ellchangecurve( $ell, v$ )`

change point z using $v = [u, r, s, t]$ `ellchangepoint( $z, v$ )`

cond, min mod, Tamagawa num $[N, v, c]$ `ellglobalred( $ell$ )`

Kodaira type of p fiber of E `ellllocalred( $ell, p$ )`

add points $z_1 + z_2$ `elladd( $ell, z_1, z_2$ )`

subtract points $z_1 - z_2$ `ellsub( $ell, z_1, z_2$ )`

compute $n \cdot z$ `ellpow( $ell, z, n$ )`

check if z is on E `ellisoncurve( $ell, z$ )`

order of torsion point z `ellorder( $ell, z$ )`

torsion subgroup with generators `elltors( $ell$ )`

y -coordinates of point(s) for x `ellordinate( $ell, x$ )`

canonical bilinear form taken at z_1, z_2 `ellbil( $ell, z_1, z_2$ )`

canonical height of z `ellheight( $ell, z, flag$ )`

height regulator matrix for pts in x `ellheightmatrix( $ell, x$ )`

p th coeff a_p of L -function, p prime `ellap( $ell, p$ )`

k th coeff a_k of L -function `ellak( $ell, k$ )`

vector of first n a_k 's in L -function `ellan( $ell, n$ )`

$L(E, s)$, set $A \approx 1$ `elllseries( $ell, s, \{A\}$ )`

root number for $L(E, \cdot)$ at p `ellrootno( $ell, \{p\}$ )`

modular parametrization of E `elltaniyama( $ell$ )`

point $[\wp(z), \wp'(z)]$ corresp. to z `ellztopoint( $ell, z$ )`

complex z such that $p = [\wp(z), \wp'(z)]$ `ellpointtoz( $ell, p$ )`

Elliptic & Modular Functions

arithmetic-geometric mean `agm( $x, y$ )`

elliptic j -function $1/q + 744 + \dots$ `ellj( $x$ )`

Weierstrass σ function `ellsigma( $ell, z, flag$ )`

Weierstrass $\wp$ function `ellwp( $ell, \{z\}, flag$ )`

Weierstrass ζ function `ellzeta( $ell, z$ )`

modified Dedekind η func. $\prod(1 - q^n)$ `eta( $x, flag$ )`

Jacobi sine theta function `theta( $q, z$ )`

k-th derivative at $z=0$ of `thetanullk( $q, k$ )`

Weber's f functions `weber( $x, flag$ )`

Riemann's zeta $\zeta(s) = \sum n^{-s}$ `zeta( $s$ )`

Graphic Functions

crude graph of $expr$ between a and b `plot( $X = a, b, expr$ )`

High-resolution plot (immediate plot)

plot $expr$ between a and b `plotoh( $X = a, b, expr, flag, \{n\}$ )`

plot points given by lists lx, ly `plotdraw( $lx, ly, flag$ )`

terminal dimensions `plotsizes()`

Rectwindow functions

init window w , with size x, y `plotinit( $w, x, y$ )`

erase window w `plotkill( $w$ )`

copy w to w_2 with offset (dx, dy) `plotcopy( $w, w_2, dx, dy$ )`

scale coordinates in w `plotscale( $w, x_1, x_2, y_1, y_2$ )`

`plotoh` in w `plotrecth( $w, X = a, b, expr, flag, \{n\}$ )`

`plotdraw` in w `plotrecthdraw( $w, data, flag$ )`

draw window w_1 at $(x_1, y_1), \dots$ `plotdraw( $[[w_1, x_1, y_1], \dots]$ )`

Low-level Rectwindow Functions

set current drawing color in w to c `plotcolor( $w, c$ )`

current position of cursor in w `plotcursor( $w$ )`

write s at cursor's position `plotstring( $w, s$ )`

move cursor to (x, y) `plotmove( $w, x, y$ )`

move cursor to $(x + dx, y + dy)$ `plotrmove( $w, dx, dy$ )`

draw a box to (x_2, y_2) `plotbox( $w, x_2, y_2$ )`

draw a box to $(x + dx, y + dy)$ `plotrbox( $w, dx, dy$ )`

draw polygon `plotlines( $w, lx, ly, flag$ )`

draw points `plotpoints( $w, lx, ly$ )`

draw line to $(x + dx, y + dy)$ `plotrline( $w, dx, dy$ )`

draw point $(x + dx, y + dy)$ `plotrpoint( $w, dx, dy$ )`

Postscript Functions

as `plotoh` `psplotoh( $X = a, b, expr, flag, \{n\}$ )`

as `plotdraw` `psplotdraw( $lx, ly, flag$ )`

as `plotdraw` `psdraw( $[[w_1, x_1, y_1], \dots]$ )`

Binary Quadratic Forms

create $ax^2 + bxy + cy^2$ (distance d) `qfb( $a, b, c, \{d\}$ )`

reduce x ($s = \sqrt{D}, l = \lfloor s \rfloor$) `qfbred( $x, flag, \{D\}, \{l\}, \{s\}$ )`

composition of forms $x*y$ or `qfbnucomp( $x, y, l$ )`

n -th power of form x^n or `qfbnupow( $x, n$ )`

composition without reduction `qfbcompraw( $x, y$ )`

n -th power without reduction `qfbpowraw( $x, n$ )`

prime form of disc. x above prime p `qfbprimeform( $x, p$ )`

class number of disc. x `qfbclassno( $x$ )`

Hurwitz class number of disc. x `qfbhclassno( $x$ )`

Quadratic Fields

quadratic number $\omega = \sqrt{x}$ or $(1 + \sqrt{x})/2$ `quadgen( $x$ )`

minimal polynomial of ω `quadpoly( $x$ )`

discriminant of $\mathbf{Q}(\sqrt{D})$ `quaddisc( $x$ )`

regulator of real quadratic field `quadregulator( $x$ )`

fundamental unit in real $\mathbf{Q}(x)$ `quadunit( $x$ )`

class group of $\mathbf{Q}(\sqrt{D})$ `quadclassunit( $D, flag, \{t\}$ )`

Hilbert class field of $\mathbf{Q}(\sqrt{D})$ `quadhilbert( $D, flag$ )`

ray class field modulo f of $\mathbf{Q}(\sqrt{D})$ `quadrday( $D, f, flag$ )`

General Number Fields: Initializations

A number field K is given by a monic irreducible $f \in \mathbf{Z}[X]$.

init number field structure nf `nfinit( $f, flag$ )`

nf members:

polynomial defining nf , $f(\theta) = 0$	<code>nf.pol</code>
number of real/complex places	<code>nf.r1, nf.r2</code>
discriminant of nf	<code>nf.disc</code>
T_2 matrix	<code>nf.t2</code>
vector of roots of f	<code>nf.roots</code>
integral basis of $\mathbf{Z}_K$ as powers of θ	<code>nf.zk</code>
different	<code>nf.diff</code>
codifferent	<code>nf.codiff</code>

recompute nf using current precision `nfnewprec( $nf$ )`

init relative rmf given by $g = 0$ over K `rmfinit( $nf, g$ )`

init bnf structure `bnfinit( $f, flag$ )`

bnf members: same as nf , plus

underlying nf	<code>bnf.nf</code>
classgroup	<code>bnf.clgp</code>
regulator	<code>bnf.reg</code>
fundamental units	<code>bnf.fu</code>
torsion units	<code>bnf.tu</code>
$[tu, fu]$	<code>bnf.tufu</code>

compute a bnf from small bnf `bnfmake( $sbnf$ )`

add S -class group and units, yield bnf s `bnfsunit( $nf, S$ )`

init class field structure bnr `bnrinit( $bnf, m, flag$ )`

bnr members: same as bnf , plus

underlying bnf	<code>bnr.bnf</code>
structure of $(\mathbf{Z}_K/m)^*$	<code>bnr.zkst</code>

Simple Arithmetic Invariants (nf)

Elements are rational numbers, polynomials, polmods, or column vectors (on integral basis $nf.zk$).

integral basis of field def. by $f = 0$ **nfbasis**(f)
field discriminant of field $f = 0$ **nfdisc**(f)
reverse polmod $a = A(X) \bmod T(X)$ **modreverse**(a)
Galois group of field $f = 0$, $\deg f \leq 11$ **polgalois**(f)
smallest poly defining $f = 0$ **polredabs**($f, flag$)
small polys defining subfields of $f = 0$ **polred**($f, flag, \{p\}$)
small polys defining suborders of $f = 0$ **polredord**(f)
poly of degree $\leq k$ with root $x \in \mathbf{C}$ **algdep**(x, k)
small linear rel. on coords of vector x **lindep**(x)
are fields $f = 0$ and $g = 0$ isomorphic? **nfisism**(f, g)
is field $f = 0$ a subfield of $g = 0$? **nfisincl**(f, g)
compositum of $f = 0$, $g = 0$ **polcompositum**($f, g, flag$)
basic element operations (prefix **nfelt**):

(**nfelt**)**mul**, **pow**, **div**, **diveuc**, **mod**, **divrem**, **val**
express x on integer basis **nfalgtobasis**(nf, x)
express element x as a polmod **nfbasistoalg**(nf, x)
quadratic Hilbert symbol (at p) **nfhilbert**($nf, a, b, \{p\}$)
roots of g belonging to nf **nfroots**($\{nf\}, g$)
factor g in nf **nfactor**(nf, g)
factor g mod prime pr in nf **nfactormod**(nf, g, pr)
number of roots of unity in nf **nfrootsof1**(nf)
conjugates of a root θ of nf **nfgaloisconj**($nf, flag$)
apply Galois automorphism s to x **nfgaloisapply**(nf, s, x)
subfields (of degree d) of nf **nfsubfields**($nf, \{d\}$)

Dedekind Zeta Function ζ_K

ζ_K as Dirichlet series, $N(I) < b$ **dirzetak**(nf, b)
init nfz for field $f = 0$ **zetakinit**(f)
compute $\zeta_K(s)$ **zetak**($nfz, s, flag$)
Artin root number of K **bnrrootnumber**($bnr, chi, flag$)

Class Groups & Units (bnf, bnr)

$a_1, \{a_2\}, \{a_3\}$ usually $bnr, subgp$ or $bnf, module, \{subgp\}$
remove GRH assumption from bnf **bnfcertify**(bnf)
expo. of ideal x on class gp **bnfisprincipal**($bnf, x, flag$)
expo. of ideal x on ray class gp **bnrisprincipal**($bnr, x, flag$)
expo. of x on fund. units **bnfisunit**(bnf, x)
as above for S -units **bnfissunit**($bnfs, x$)
fundamental units of bnf **bnfunit**(bnf)
signs of real embeddings of $bnf.fu$ **bnfsignunit**(bnf)

Class Field Theory

ray class group structure for mod. m **bnrclass**($bnf, m, flag$)
ray class number for mod. m **bnrclassno**(bnf, m)
discriminant of class field ext **bnrdisc**($a_1, \{a_2\}, \{a_3\}$)
ray class numbers, l list of mods **bnrclassnolist**(bnf, l)
discriminants of class fields **bnrdisclist**($bnf, l, \{arch\}, flag$)
decode output from **bnrdisclist** **bnfdecodemodule**(nf, fa)
is modulus the conductor? **bnrisconductor**($a_1, \{a_2\}, \{a_3\}$)
conductor of character chi **bnrconductorofchar**(bnr, chi)
conductor of extension **bnrconductor**($a_1, \{a_2\}, \{a_3\}, flag$)
conductor of extension def. by g **rnfconductor**(bnf, g)
Artin group of ext. def'd by g **rnfnormgroup**(bnr, g)
subgroups of bnr , index $\leq b$ **subgrouplist**($bnr, b, flag$)
rel. eq. for class field def'd by sub **rnfkummer**($bnr, sub, \{d\}$)
same, using Stark units (real field) **bnrstark**($bnr, sub, flag$)

PARI-GP Reference Card (2)

(PARI-GP version 2.3.0)

Ideals

Ideals are elements, primes, or matrix of generators in HNF.
is id an ideal in nf ? **nfisideal**(nf, id)
is x principal in bnf ? **bnfisprincipal**(bnf, x)
principal ideal generated by x **idealprincipal**(nf, x)
principal idele generated by x **ideleprincipal**(nf, x)
give $[a, b]$, s.t. $a\mathbf{Z}_K + b\mathbf{Z}_K = x$ **idealtwoelt**($nf, x, \{a\}$)
put ideal a ($a\mathbf{Z}_K + b\mathbf{Z}_K$) in HNF form **idealhnf**($nf, a, \{b\}$)
norm of ideal x **idealnrm**(nf, x)
minimum of ideal x (direction v) **idealmin**(nf, x, v)
LLL-reduce the ideal x (direction v) **idealred**($nf, x, \{v\}$)

Ideal Operations

add ideals x and y **idealadd**(nf, x, y)
multiply ideals x and y **idealmul**($nf, x, y, flag$)
intersection of ideals x and y **idealintersect**($nf, x, y, flag$)
 n -th power of ideal x **idealpow**($nf, x, n, flag$)
inverse of ideal x **idealinv**(nf, x)
divide ideal x by y **idealdiv**($nf, x, y, flag$)
Find $(a, b) \in x \times y$, $a + b = 1$ **idealaddtoone**($nf, x, \{y\}$)

Primes and Multiplicative Structure

factor ideal x in nf **idealfactor**(nf, x)
recover x from its factorization in nf **factorback**(x, nf)
decomposition of prime p in nf **idealprimedec**(nf, p)
valuation of x at prime ideal pr **idealval**(nf, x, pr)
weak approximation theorem in nf **idealchinese**(nf, x, y)
give bid = structure of $(\mathbf{Z}_K/id)^*$ **idealstar**($nf, id, flag$)
discrete log of x in $(\mathbf{Z}_K/bid)^*$ **ideallog**(nf, x, bid)
idealstar of all ideals of norm $\leq b$ **ideallist**($nf, b, flag$)
add archimedean places **ideallistarch**($nf, b, \{ar\}, flag$)
init **prmod** structure **nfmodprinit**(nf, pr)
kernel of matrix M in $(\mathbf{Z}_K/pr)^*$ **nfkermodpr**($nf, M, prmod$)
solve $Mx = B$ in $(\mathbf{Z}_K/pr)^*$ **nfsolvemodpr**($nf, M, B, prmod$)

Galois theory over $\mathbf{q}$

initializes a Galois group structure **galoisinit**($pol, \{den\}$)
action of p in **nfgaloisconj** form **galoispermopol**($G, \{p\}$)
identifies as abstract group **galoisidentify**(G)
exports a group for GAP or MAGMA **galoisexport**($G, flag$)
subgroups of the Galois group G **galoissubgroups**(G)
subfields from subgroups of G **galoissubfields**($G, flag, \{v\}$)
fixed field **galoisfixedfield**($G, perm, flag, \{v\}$)
is G abelian? **galoisisabelian**($G, flag$)
abelian number fields **galoissubcyclo**($N, H, flag, \{v\}$)

Relative Number Fields (rnf)

Extension L/K is defined by $g \in K[x]$. We have $order \subset L$.
absolute equation of L **rnfequation**($nf, g, flag$)
relative **nfalgtobasis** **rnfalgtobasis**(rnf, x)
relative **nfbasistoalg** **rnfbasistoalg**(rnf, x)
relative **idealhnf** **rnfidealhnf**(rnf, x)
relative **idealmul** **rnfidealmul**(rnf, x, y)
relative **idealtwoelt** **rnfidealtwoelt**(rnf, x)

Lifts and Push-downs

absolute $\rightarrow$ relative repres. for x **rnfeltabstorel**(rnf, x)
relative $\rightarrow$ absolute repres. for x **rnfeltreltoabs**(rnf, x)
lift x to the relative field **rnfeltup**(rnf, x)
push x down to the base field **rnfeltdown**(rnf, x)
idem for x ideal: (**rnfideal**)**reltoabs**, **abstorel**, **up**, **down**

Projective $\mathbf{Z}_K$ -modules, maximal order

relative **polred** **rnfpolred**(nf, g)
relative **polredabs** **rnfpolredabs**(nf, g)
characteristic poly. of $a \bmod g$ **rnfcharpoly**($nf, g, a, \{v\}$)
relative Dedekind criterion, prime pr **rnfdedekind**(nf, g, pr)
discriminant of relative extension **rnfdisc**(nf, g)
pseudo-basis of $\mathbf{Z}_L$ **rnfpseudobasis**(nf, g)
relative HNF basis of $order$ **rnfhnfbasis**($bnf, order$)
reduced basis for $order$ **rnflllgram**($nf, g, order$)
determinant of pseudo-matrix A **rnfdet**(nf, A)
Steinitz class of $order$ **rnfsteynitz**($nf, order$)
is $order$ a free $\mathbf{Z}_K$ -module? **rnfisfree**($bnf, order$)
true basis of $order$, if it is free **rnfbasis**($bnf, order$)

Norms

absolute norm of ideal x **rnfidealnrmabs**(rnf, x)
relative norm of ideal x **rnfidealnrmrel**(rnf, x)
solutions of $N_{K/\mathbf{Q}}(y) = x \in \mathbf{Z}$ **bnfisintnorm**(bnf, x)
is $x \in \mathbf{Q}$ a norm from K ? **bnfisnorm**($bnf, x, flag$)
initialize T for norm eq. solver **rnfisnorminit**($K, pol, flag$)
is $a \in K$ a norm from L ? **rnfisnorm**($T, a, flag$)

Based on an earlier version by Joseph H. Silverman

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